Lecture 11 - Discrete Q-learning

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DDA4230: Reinforcement Learning
Course Page: [Click]

Model-based v.s. Model-free Algorithms

The model indicates the transition function and the reward function. This estimation could be in form of point estimation or distribution estimation like posterior sampling.

- Model-based Algorithm: maintains an estimate of the model and uses the model when interacting with the environment.
- Model-free Algorithm: does not estimate the world model.

When we do not have a reasonable estimation of the model (under large state and action spaces and continuous settings), an error will be induced by a wrongly estimated model as the model bias (maybe accumulate during learning).

Q-Learning

We start with the value iteration algorithm and discuss how the model could be lifted.

Algorithm 1: Value iteration

Input: ϵ

For all states $s \in S$, $V'(s) \leftarrow 0$, $V(s) \leftarrow \infty$

while $\|V - V'\|_\infty > \epsilon$ **do**

  $V \leftarrow V'$

  For all states $s \in S$, $V'(s) = \max\limits_{a \in A} \left[R(s,a) + \gamma \sum_{s' \in S} P(s' \mid s,a) V(s')\right]$

$V^* \leftarrow V$ for all $s \in S$

$\pi^* \leftarrow \arg\max\limits_{a \in A} \left[R(s,a) + \gamma \sum_{s' \in S} P(s' \mid s,a) V^*(s')\right] \quad , \forall\, s \in S$

return $V^*(s)$, $\pi^*(s)$ for all $s \in S$

Q-Learning

- The terms $\sum_{s' \in S} P_{\mathcal{T}}(s' \mid s, a) V(s')$ and $\sum_{s' \in S} P_{\mathcal{T}}(s' \mid s, a) V^*(s')$ could remove the dependency on $P_{\mathcal{T}}$ by representing the action values.
- $V'(s) = \max_{a \in A}[R(s, a) + \gamma \sum_{s' \in S} P_{\mathcal{T}}(s' \mid s, a) V(s')]$ can be updated to $Q'(s, a) = \max_{a' \in A}[R(s, a) + \gamma \sum_{s' \in S} P_{\mathcal{T}}(s' \mid s, a)[Q(s', a')]]$ (Free R and $P_{\mathcal{T}}$).

Algorithm 2: Q-learning

Input: ϵ, α

For all $(s, a) \in \mathcal{S} \times \mathcal{A}$, $Q'(s, a) \leftarrow 0$, $Q(s, a) \leftarrow \infty$

while $\|Q - Q'\|_\infty > \epsilon$ **do**

  $Q \leftarrow Q'$

  Sample a trajectory τ from the policy $\pi(a \mid s) = \arg\max_{a \in A} Q(s, a)$

  For all state-action-reward-state tuple $(s, a, r, s') \in \tau$,

   $Q'(s, a) \leftarrow (1 - \alpha) Q(s, a) + \alpha \max_{a' \in \mathcal{A}} [r + \gamma Q(s', a')]$

$Q^* \leftarrow Q$ for all $(s, a) \in \mathcal{S} \times \mathcal{A}$

$\pi^* \leftarrow \arg\max_{a \in \mathcal{A}} Q(s, a)$

return $Q^*(s, a)$, $\pi^*(s)$ for all s, a

Q-Learning

- Introducing the step size so that the update only takes at α portion of the action value while the $1-\alpha$ portion of the action value remains the same.

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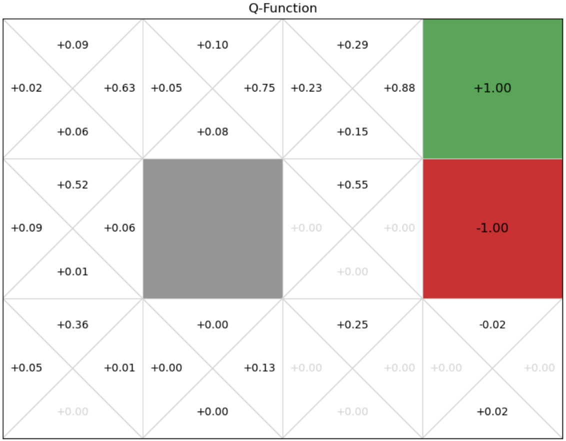
return $Q^*(s,a)$, $\pi^*(s)$ for all s, a

Q-Learning

Q Learning in Grid World.



Exploration and ε-greedy Q-learning

In Q-learning, the trajectory sampled is subject to the current policy and thereof the current value estimation. However,

- It is possible that the algorithm is stuck at a suboptimal action value estimate and does not update itself.
- It is possible that some states are never explored with some initialization of the policy and value functions.

A simple way of involving exploration is to force the algorithm to select a random action with probability ε. This ε could delay over the iterations, as is in the ε-greedy algorithm for multi-armed bandits.

Exploration and ε-greedy Q-learning

Algorithm 3: Q-learning with ε-greedy exploration

Input: ϵ, α

For all $(s, a) \in \mathcal{S} \times \mathcal{A}$, $Q'(s, a) \leftarrow 0$, $Q(s, a) \leftarrow \infty$

while $\|Q - Q'\|_\infty > \epsilon$ **do**

  $Q \leftarrow Q'$

  Sample a trajectory τ from the policy

$$\pi(a \mid s) = \begin{cases} \arg\max_{a \in A} Q(s, a) & \text{with probability } 1 - \varepsilon \\ \text{random action} & \text{with probability } \varepsilon \end{cases}$$

  For all state-action-reward-state tuple $(s, a, r, s') \in \tau$,

  $Q'(s, a) \leftarrow (1 - \alpha) Q(s, a) + \alpha \max_{a' \in \mathcal{A}} [r + \gamma Q(s', a')]$

$Q^* \leftarrow Q$ for all $(s, a) \in \mathcal{S} \times \mathcal{A}$

$\pi^* \leftarrow \arg\max_{a \in \mathcal{A}} Q(s, a)$

return $Q^*(s, a)$, $\pi^*(s)$ for all s, a

Q-learning with UCB

In spite of simplicity, ε-greedy Q-learning does not have a rigorous regret guarantee.

- We present another variant of Q-learning with UCB exploration. This algorithm is the first Q-learning variant that has a rigorous regret guarantee of $\sqrt{K}$.
- We again use $Q_h(s,a)$ as the time-dependent action-value function, which is necessary when the horizon of each episode is constant.

Q-learning with UCB

Algorithm 4: Q-learning with UCB exploration

Input: α: adaptive step size; δ: confidence level

Initialize $Q_h(s,a) \leftarrow 0$, $N_h(s,a) \leftarrow 0$ for all $h \in [H]$, $k \leftarrow 0$

while $k \leq K-1$ **do**

  Start an episode with s_0

  for $h \leq H-1, \dots, 0$ **do**

    Take action $a_h^k = \arg\max_a Q_h(s_h^k, a)$ and observe s_{h+1}^k

    $N_h(s_h^k, a_h^k) \leftarrow N_h(s_h^k, a_h^k) + 1$

    Update the action value as

$$Q_h(s_h^k, a_h^k) \leftarrow (1-\alpha) Q_h(s_h^k, a_h^k) + \alpha \left[r_h(s_h^k, a_h^k) + V_{h+1}(s_{h+1}^k) + c\sqrt{\frac{H^3 \log(nmHK/\delta)}{N_h(s_h^k, a_h^k)}} \right]$$

    Update the state value as

$$V_h(s_h^k) = \min\left\{ \max_a Q_h(s_h^k, a), H \right\}$$

  $k \leftarrow k+1$

$Q_h^* \leftarrow Q_h$

$\pi_h^* \leftarrow \arg\max_a Q_h(s,a)$

return Q_h^*, π_h^* for all $h \in [H]$

Q-learning with UCB

Theorem

By choosing $\alpha = \frac{H+1}{H+N}$ *with the visitation count* $N = N_h(s_h^k, a_h^k)$*, there exists an absolute constant c such that with probability at least* $1-\delta$ *the regret of Q-learning with UCB exploration is at most* $O(\sqrt{nmH^5 K \log(nmHK/\delta)})$.

The proof relies on the cast of the variables into a filtration and therefore the use of the Azuma-Hoeffding inequality (introduced in LN3). For those students that are interested in the proof we could host you with a presentation of it.

Question and Answering (Q&A)